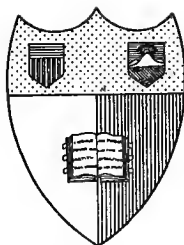


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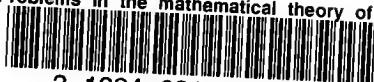
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PROBLEMS IN THE MATHEMATICAL THEORY OF INVESTMENT

BY

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PREFATORY NOTE

When my book on "The Mathematical Theory of Investment" was written, considerable difficulty was experienced in finding a sufficient number of practical problems with which to illustrate the theory. As every teacher of elementary mathematics knows, solutions of the problems in any book that is used from year to year soon become current among the student body, so that the usefulness of a limited book list is to a large extent destroyed.

To remedy this difficulty the following list of one hundred problems, based upon a common fund of material accumulated by Dr. G. R. Clements, Professor Arnold Dresden, Dr. Florence E. Allen, and Dr. T. M. Simpson, of the University of Wisconsin, has been prepared by Dr. Clements. The problems are simple in content and in the amount of computation required. They are not closely graded, but the student is required to determine for himself the class into which each problem falls and to select the formula required for its solution. It is believed that the list will be of real assistance to teachers who wish to obtain the best possible results from classes studying the theory of investment.

E. B. SKINNER

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PROBLEMS IN THE MATHEMATICAL THEORY OF INVESTMENT

The analysis of a problem.

The problems in this list have to do with

I. A *single payment* (or several payments irregular either in amount or in frequency of payment); or

II. An *annuity*, that is, a series of payments equal in amount, made at regular intervals of time.

I. *Single payments* are concerned with

P (the principal), the value of the transaction at its beginning ;

n , the time (in years);

i , the total annual interest return on a unit of principal; or

j , the nominal rate of interest, and

m , the conversion frequency of interest into principal ;

S , the value of the transaction at its close.

These quantities satisfy the relations

$$S = P(1 + i)^n,$$
$$1 + i = \left(1 + \frac{j}{m}\right)^m.$$

(If simple interest is used, $S = P(1 + ni)$.)

II. *Annuities* are concerned with

R , the annual rent of the annuity ;

n , the term of the annuity (in years);

p , the payment frequency ;

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i (or j , convertible m times a year), the rate of interest;
 A , the sum of the present values obtained by discounting
 each annuity payment to the beginning of the term;
 K , the sum of the amounts obtained by accumulating each
 annuity payment to the close of the term.

If n is finite, these quantities satisfy the relations

$$A = Ra_{\overline{n}|}^{(p)},$$

$$K = Rs_{\overline{n}|}^{(p)},$$

and if n is infinite,

$$A_{\infty} = Ra_{\infty, k} = \frac{R}{i} \cdot \frac{1}{s_{\overline{k}|}},$$

where k is the payment frequency of the perpetuity.

In making computations by means of available tables the following formulas are used:

$$\frac{1}{s_{\overline{n}|}} = \frac{1}{a_{\overline{n}|}} - i,$$

$$s_{\overline{n}|}^{(p)} = s_{\overline{n}|} \frac{i}{j_{(p)}}, \quad a_{\overline{n}|}^{(p)} = a_{\overline{n}|} \frac{i}{j_{(p)}};$$

and, in general, when $m \neq p$, multiplication of both numerator and denominator of $s_{\overline{n}|}^{(p)}$ and $a_{\overline{n}|}^{(p)}$ by i or by j/m transforms them into the product of quantities listed in the tables.

Equation of value. To compare two plans for meeting the same set of obligations, accumulate and discount each to the same date at a specified rate of interest, usually the current rate. To say that the two plans are equivalent means that the two amounts thus determined must be

equal. A statement of their equality is called an *equation of value*.

Clearness can frequently be gained by stating briefly in parallel columns the plans to be compared.

To analyze a problem

- (a) separate the problem into parts which relate to I or to II;
- (b) make a list of the items known and of those to be determined ;
- (c) substitute the known numbers in the simplest appropriate formulas, and if possible solve the equations thus set up for the numbers to be determined, or
- (c') substitute in the appropriate formulas to determine the value of each transaction on a specified date, set up an equation of value, and solve this equation for the number to be determined.

It is highly desirable that the student be able to prove the laws listed above and, on the basis of definitions, to derive from them, by an analysis such as that just outlined, the discussion of such topics as equation of payments, amortization of debts, sinking funds and depreciation funds, capitalized cost, extension of life, composite life, valuation of bonds, etc. It will prove desirable at times to introduce additional formulas for the sake of ease in computation, but it will tend to clarity if one recognizes that these few simple relations and the fundamental principle of the *equation of value* supply a connecting thread which binds all together.

First, *analyze the problem*. Get the situation clearly before you ; see precisely what is given and what is to be determined before you think of a formula. The appropriate formula will then be obvious.

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PROBLEMS

Solve problems 1-4 as geometrical progressions.

1. The amount of coal burned during the year 1907 is estimated at 480,363,424 tons. The normal increase in consumption is estimated to be 7.36% per annum. Find the amount that will be consumed in 1920 if this rate of increase is maintained (see Skinner, *Mathematical Theory of Investment*, p. 67).

2. If A tons of coal are burned in a given year, and if the fuel consumption increases uniformly at rate i per year, how much is burned in the n th year following? How much is burned in n years? In how many years will B tons be consumed?

3. If there are \$100 in a fund, and if the fund is increased at the end of each year by 6% of the amount in the fund during that year, to how much will it accumulate by the end of five years?

4. If there are P dollars in a fund, and if the fund is increased at the end of each of n periods by the fraction i of the amount in the fund during that period, to how much will the fund accumulate by the end of the n th period?

5. A merchant is offered the option of paying his bill in thirty days or of receiving 2% discount for cash in ten days. At what rate of interest can he afford to borrow money and get the discount?

6. An investment of \$500 yields 5% annual return. Find the value of this investment at the end of four years.

7. When a boy is born, \$100 is placed to his credit in a savings bank that pays 4% nominal, convertible semiannually. If the account is not disturbed until the boy's nineteenth birthday, how much will there be to his credit?

8. A woman who has funds on deposit in a savings bank that pays 4% nominal, convertible quarterly, is considering

the purchase of municipal bonds which can be bought to yield 4.15% annually. Is it to her advantage to change the form of her investment? What is the difference in annual return on an investment of \$10,000?

9. At what rate will a sum of money treble itself in fifteen years, interest converted quarterly?

10. A man has \$600 invested for five years at 5% nominal, payable semiannually. What sum could he afford to accept in settlement of this account if he could reinvest his funds for the same period at 6% effective? Restate this problem in terms of present value.

11. Would it be more profitable to invest money at $5\frac{1}{4}\%$, payable annually, or at a nominal rate of 5%, payable quarterly? What nominal rate, convertible quarterly, would be exactly equivalent to a rate of $5\frac{1}{4}\%$, payable annually?

12. On September 1, 1914, a student borrows \$500 and gives his note promising to repay this sum, together with the interest thereon at 5%, compounded annually, on September 1, 1919. On September 1, 1916, this note is bought to yield the purchaser 6% on his investment. How much does he pay?

13. At what nominal rate, convertible quarterly, will \$500 accumulate to \$673.43 in five years? Find the corresponding effective rate of interest.

14. A grocer receives a bill of goods from his wholesaler on which he is offered 30 days' time in which to pay, or 2% discount for cash. What rate of interest can the grocer afford to pay his banker in order to borrow money and get the discount?

15. If money is worth 5%, what equal amounts, payable in one and in two years respectively, could fairly replace three notes given on the following terms: \$500 due in one year, with interest at 5%; \$1000 due in three years, with interest at 6%; and \$400 due in two and one-half years, with interest at 5%?

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16. In what time could debts of \$200, \$450, and \$1000, due in two, one, and three years respectively, be fairly replaced by a single payment of \$1650 if money is worth 6% effective?

17. A father has three children, aged four, seven, and nine years. He wishes to present each child with \$400 when he has reached the age of twenty-one. How much should he lay aside now for that purpose if money can be accumulated at 4% effective?

18. If a pawnbroker keeps his capital invested at 3% per month, find his annual income per unit of investment.

19. A man buys a farm for \$5000. He pays \$1000 cash and agrees to pay the balance, principal and interest at 6%, in three equal annual installments. Find the annual payment.

20. A benevolent organization is offered \$3000, with the proviso that it shall pay the donor 6% thereon during the rest of her life. If the association can borrow money and invest its funds at 5%, show that it would be to its advantage to accept this offer if payments may reasonably be expected to continue for not more than thirty-six years.

21. A clerk expects to go into business for himself as soon as he has saved \$5000. He now has \$2000. If he can invest all his funds at 5% effective, how much must he save per annum in order to go into business at the end of five years?

22. How many years will it take to accumulate \$1000 by investing \$100 per annum at 6% effective?

23. Which is worth more, the expectation of an annual income of \$500 a year for twelve years, first payment made one year hence, or one of \$600 a year for ten years, first payment made three years hence, if money is worth 5%?

24. A city issues bonds for \$100,000, on which it pays interest at 5% per annum. It accumulates a sinking fund at 4% to retire the bonds at the end of fifteen years. Find the amount to be provided annually.

25. What sum must be put semiannually into an investment that pays 5% effective in order to accumulate \$1000 in five years?

26. A person invests \$5000 at 5% effective. Principal and interest are to provide a fixed annual income for twenty years, at the end of which time the capital is to be exhausted. The first payment is to be made one year from date. How large an income does he receive?

27. A ring is offered for \$20 cash and \$10 at the end of each month for eight months. Find its cash value if money is worth 6% nominal, convertible monthly. Use the binomial theorem in making the computation.

NOTE. Reputable credit houses that make the offer given above usually offer 10% discount for cash; that is, on the interest basis here assumed, they charge \$8.23 for the privilege of credit.

28. A set of books is offered for \$166.75 cash, or for \$5 cash and \$5 per month for thirty-six months. Compare the offers when money is worth 6% effective.

NOTE. The second offer has a present value of \$169.74; that is, the selling company is charging only \$3 for carrying this account for three years. On a basis of 6% nominal, convertible monthly, the present value of the installment plan is \$169.38.

29. A mortgage for \$5000 is given on July 1, 1912, to be repaid, principal and interest at 6% effective, in ten equal semiannual installments, the first to be made January 1, 1913. Find the size of the semiannual payment.

On July 1, 1915, this mortgage is sold to yield the purchaser 5% effective on his investment. How much does he pay for the mortgage?

30. How much should one put aside at the birth of a son to be able to furnish him with \$500 during each of four college years, beginning at the age of nineteen? Assume that money will be worth 4% throughout the life of this fund.

31. According to the conditions of a will a sum of \$18,000 is to be held in trust until it has accumulated to \$25,000. If funds can be accumulated at 5%, convertible semiannually, when will the beneficiary receive control of his money?

32. A father bequeaths to a son who is just fifteen years old ten thousand dollars' worth of a preferred stock which pays dividends at the rate of 6% nominal, payable quarterly. The will further directs that this stock and the profits from it shall be held in trust until the boy is twenty-five years old. Assuming that the dividend rate continues unchanged, and that the income is kept invested at 5% effective, find the value of this property on the beneficiary's twenty-fifth birthday.

33. A corporation issues 1000 bonds of \$100 each, to be redeemed at par in twenty years, and bearing interest at 4% effective. How much must be provided annually to pay the interest and to provide a sinking fund for redeeming the bonds at maturity, if the sinking fund can be accumulated at 4%? How much would the net indebtedness of the corporation be immediately after the eighth payment had been made?

34. Which is the more rapid way to accumulate \$4000 — to deposit, in a bank that pays 4% effective, \$100 every year, beginning one year from date, or to deposit \$1500 in the same bank now?

35. Derive the formula for the present value of a perpetuity of 1, payable annually (or every k years), as the endowment on which the interest at rate i per annum will be 1 every year (or every k years).

36. Derive the formula for the present value of a perpetuity of 1, payable every k years, as the present value of a perpetuity of $1/s_k$, payable annually. Explain in detail.

37. How large an endowment is necessary for a room in a hospital, costing \$500 to install and \$160 annually to maintain, if money can be invested at 4% effective?

38. Prove that the capitalized cost of a depreciable article costing C , and having to be replaced at the initial cost every k years, is $C_\infty = \frac{C}{i} \cdot \frac{1}{a_{\overline{k}|i}}$ (see Skinner, p. 89, formula (4)).

39. Is it more profitable for a city to pay \$2 per square yard for paving which lasts five years or to pay \$3 per square yard for paving which lasts eight years, if money is worth 5%?

40. Prepare a schedule for the amortization of a debt of \$3000, with interest at 6% nominal, payable semiannually, by a series of ten equal semiannual payments, the first made six months after the date of the loan.

41. Prepare a schedule for the repayment of a debt of \$100,000, expressed in bonds of denomination \$100, bearing interest at 6%, by ten annual payments as nearly equal as possible.

42. Find the value, immediately after a crop has been sold, of a piece of land that can produce a crop worth \$800 every second year. Money is worth 4%.

43. A business man wishes to set aside on his fortieth birthday a sum which will provide him with an income of \$1000 a year for ten years, beginning on his sixty-first birthday. Find the sum required, if money is worth 5% effective.

44. If a corporation using auto trucks is buying tires that cost \$85 per set, net, and average four months of wear, how much can it afford to pay for better tires that will have an average life of six months, money worth 4%?

45. An issue of \$50,000 in bonds of denomination \$100, bearing interest at 5%, is secured by a mortgage on a newly constructed store building. It is planned to pay interest alone for two years, and, beginning three years from date of issue, to retire the bonds in ten years by payments, including interest, as nearly equal as possible. Construct a schedule showing the condition of the debt at the end of each year.

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46. A bond, face value \$1000, redeemable at par on January 1, 1920, and bearing interest at 6%, payable January 1 and July 1, is bought on April 1, 1914, to yield the purchaser 4%, convertible January 1 and July 1. Find the purchase price.

47. A 6% bond, face value \$100, interest payable January 1 and July 1, and redeemable at par on January 1, 1919, is bought July 1, 1916, to yield the purchaser 10%, convertible January 1 and July 1. Find the purchase price and prepare a schedule showing the book value of the bond on each dividend date.

48. A house is offered for sale for \$500 cash and \$1000 at the end of each year for five years, or for \$5000 cash. Which is better for the buyer, if he can borrow money at 5% ?

49. A manufacturing concern contracts for a factory site for \$10,000 cash and \$5000 at the end of each year for five years. If money is worth 5%, what would be a fair cash settlement ?

50. A man gives a mortgage on his home for \$4000. What sum must he provide annually to pay interest at 6% on the debt, and to provide for paying the mortgage when it matures in ten years by depositing a fixed sum each year in a savings bank that pays 4% effective ?

Would it be better to amortize this debt, with interest at 5% nominal, convertible semiannually, in equal semiannual payments for ten years ?

51. A man buys a city lot unimproved by buildings for \$3000 and five years later sells it for \$5000. At the end of each year of his ownership he pays taxes at the rate of fifteen mills per dollar of an assessed valuation which is \$2500 during the first three years of this period and \$3000 during the last two years. He pays an agent a commission of 1% for selling the lot. Compare his profits with the return from investing at 6% effective all sums he has here paid. Find the rate of interest return paid by the real-estate investment.

52. A city wishes to secure, for the purpose of establishing a park and playground, a piece of real estate that has recently been improved with buildings. The property in its present condition has a market value of \$100,000. To satisfy the condition that a bequest from which the purchase price is to be obtained must be used within one year, it is proposed that the city buy this property immediately and allow the seller to occupy it free of rent, except for the cost of necessary repairs, for ten years, at the end of which time the city is to secure full possession. If it may be assumed that money will be worth 5% per annum and the tax rate will be 16 mills for the next ten years, find a fair purchase price for this property.

53. Two hundred members of the senior class of a certain college plan to present their alma mater with a scholarship fund of \$10,000 on the tenth anniversary of their graduation. They propose to accumulate this fund by making equal payments on each commencement day, beginning one year after date of graduation. If the fund can be accumulated at 5%, find the annual payment each member must make.

54. A college is to receive \$250,000, payable at the death of a daughter of the donor. If there is no reasonable probability that the college will come into possession of this property in less than twenty years, what sum can the trustees of the college afford to accept in full settlement of their expectation from this estate, if money is worth 5%?

55. A debt of \$200,000 is to be paid in twenty years by means of a sinking fund established at the time the debt is incurred. If the sinking fund can be accumulated at 4% effective, how large will it be at the end of ten years?

How much must the debtor provide annually to maintain the sinking fund and to pay interest at 5% on the debt?

Would it be better for him to amortize the debt in equal semiannual installments for twenty years, with interest at 5% nominal, convertible semiannually?

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56. How much can one afford to expend on a piece of equipment which must be replaced at a cost of \$100 every three years for an unlimited time, in order to get better equipment which will need replacement only every five years? Assume money steadily worth 4%.

Let x = the cost of the better article.

Then we have to compare

\$100 every three years in perpetuity.		x every five years in perpetuity.
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To compare two plans we must find the value of each at the *same* time, in this case preferably at the present. Since here we must add the cost of first installation to the present value of the perpetuity, this is equivalent to finding the capitalized cost of each article. These plans will be equivalent in the long run if their capitalized costs are equal. This gives the equation of value

$$\frac{100}{.04} \cdot \frac{1}{a_{\overline{3}|}} = \frac{x}{.04} \cdot \frac{1}{a_{\overline{5}|}}, \text{ at } 4\%,$$

whence
$$x = 100 \frac{1}{a_{\overline{3}|}} a_{\overline{5}|} = 160.42.$$

One can therefore afford to expend $x - 100 = 60.42$ for the improvement of the piece of equipment in order to make it last five years instead of three.

57. Find the capitalized cost of an article costing \$10 new, that must be replaced every four years, money assumed to be steadily worth 4%. Without using any formulas except those for capitalized cost or present value of a perpetuity, find how much one could afford to expend to extend the life of this article to six years.

58. Show that the amount which one can afford to expend to replace a depreciable article costing C and needing replacement

at the initial cost every k years, by another article costing x and needing replacement every k' years, is

$$x = C \frac{1}{a_{\overline{k}|}} a_{\overline{k'}|}.$$

Note that there is no restriction on the relative values of k and k' .

59. An auto truck costing \$1800 has a probable life of four years and a scrap value of \$100. If money is worth 5%, find the theoretical second-hand value of the truck at the end of two years. (Theoretically the value of the truck, plus the value in the depreciation fund, should remain constant.)

60. A plant consists of the following items:

	COST NEW	SCRAP VALUE	LIFE
A	20,000	2000	18 years
B	11,000	1500	15 years
C	30,000	0	5 years

Find the annual depreciation charge and the composite life of the plant on a 4% basis.

61. If a plant costs \$60,000 to build, has a probable scrap value of \$10,000, and requires an annual depreciation charge of \$5600, find its composite life on a 4% basis.

62. A bank loans a farmer \$2000, to be repaid with interest at 7% payable semiannually, in equal semiannual installments for fifteen years. Find the semiannual payment.

After holding this mortgage three years the bank sells it to yield the purchaser 6% effective; find the selling price. Find the selling price to yield the purchaser 6% nominal, convertible semiannually.

63. A man has a mortgage on his home for \$10,000, to be paid at the end of ten years and bearing interest at 6%, payable annually. He wishes to establish a sinking fund by equal

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payments at the end of each year, to provide for paying the mortgage when it becomes due. If this fund can be accumulated at 5%, how much must he provide annually for interest and sinking-fund payment?

How much will be in the sinking fund at the end of six years?

Would it be better to amortize the debt with interest at 5% nominal, convertible semiannually, in equal semiannual installments?

64. What equal amounts must one deposit at the end of each month in a savings bank that pays 3% nominal, convertible semiannually, in order to buy a \$200 sailboat at the end of eighteen months?

65. A phonograph is offered for \$100 cash, or for \$4 cash and \$8 at the end of each month for twelve months. If money is worth 6% effective, and if 50¢ should be charged for opening an account, find what bonus the music company is giving for the sake of making an easier sale.

66. An industrial commission awards the wife of a workman killed in an accident compensation to the amount of \$3000, but suggests that this award be paid at the rate of \$25 per month. If money is assumed to be worth $3\frac{1}{2}\%$, for how long should these payments be continued?

67. A husband and wife give \$5000 toward the endowment of a hospital, with the provision that the fund shall be allowed to accumulate for 100 years. If the fund is kept invested at an average rate of $4\frac{1}{2}\%$, find the value of the endowment when it becomes available.

68. A man begins at the age of twenty-five to save \$10 per month, and keeps all his savings invested at an average rate of 5% effective. How much will he have when he is sixty years old?

69. If the beneficiary of a life-insurance policy for \$3000 chooses to accept in settlement ten equal annual installments,

the first to be paid one year after date of the agreement, find the annual payment, money worth 3%. Find the payment if the first installment is to be paid immediately.

70. A board walk costing \$40 must be built immediately and every eight years hereafter. How much can the owner afford to pay to build instead of this a concrete walk, assumed to be permanent, if money is worth 4%?

How much can he afford to pay on the assumption that the concrete walk will need replacing at the original cost every twenty years?

71. A savings bank receives deposits of \$7 monthly for ten years, and returns to the depositor \$1000 at the close of the ten-year period. Find the rate of interest paid, making the computation as if \$42 were deposited at the end of each six months.

72. What sum, paid at the end of each year for five years, will fairly replace debts of \$1000, \$650, and \$475, due without interest in three, one, and four years respectively, if money is worth 5%?

73. A land company is organized with a capital stock of \$100,000, all paid in. Land is purchased for \$80,000 and is subdivided and sold for \$120,000, to be paid with interest at 6% in twenty \$6000 annual payments, the first to be made in one year. The promoters of the company are to receive \$20,000 in cash, and in return are to assume all the expense involved in the promotion and conduct of the business. It is further agreed that at the end of each year the promoters are to reinvest, on these same terms, a sufficient part of the receipts to keep the amount of capital invested in the business the same from year to year. All further receipts are to be distributed to the stockholders annually. If this plan can be carried out, what rate of income will be realized by the stockholders?

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74. Which is more profitable, to rent an ocean-going freight boat for \$10,000 per six months for ten years or to buy it for \$175,000, when money is worth 4% nominal, convertible semi-annually, assuming that the boat will be worthless at the end of this time?

75. A debt of \$8475, bearing interest at 5%, is being paid, principal and interest, in equal annual installments of \$1000. In how many years will the debt be discharged? Prepare a schedule showing the condition of the debt at the end of each year and giving the amount of the irregular payment closing the transaction.

76. A man owes \$700 due in seven years without interest, \$1000 due in one year with interest at $4\frac{1}{2}\%$ effective, and \$400 due in five years with interest at 6% nominal, convertible semiannually. What single payment made two years from date will discharge all these debts if money is worth 4% effective? What payments equal in amount made at the end of each year for three years would fairly replace these debts?

77. A bond of face value \$100, bearing interest at 5%, payable January 1 and July 1, and redeemable at \$105 on January 1, 1921, is bought on January 1, 1917, to yield the purchaser 8% nominal, convertible semiannually. Find the purchase price and construct a schedule for the accumulation of the discount.

78. How much must a philanthropist give to purchase and maintain indefinitely an automobile ambulance which costs \$2000, if the annual operating expense is \$1000 and if the car must be replaced every six years, with a scrap value of \$300? Assume that money can be invested at 5%.

79. A real-estate company offers to sell a lot and loan the purchaser the money with which to build a house, provided he makes a cash payment of 10% of the combined cost of house and lot, and makes monthly payments of 1% of this

cost until the total amount is paid. If money is worth 6% nominal, convertible monthly, how long will it take to discharge the debt?

80. A garden hose costs \$10 and will last three years. How much can one afford to pay for a better grade of hose that will last for five years, money worth 4%?

81. If a silo costs a farmer \$200 and saves him \$10 a month on his feed bills, how long will it take for the silo to pay for itself, money worth 6% per annum?

82. A man mortgages his home for \$5000 and arranges to amortize the debt, with interest at 6%, in ten equal annual payments. What sum would complete the payment of the debt immediately after the sixth payment has been made?

83. If a man has a salary of \$125 per month, what per cent of his salary must he put monthly into an investment that pays 5% annually in order to save \$1000 in four years?

84. A farm yields an average annual crop worth \$2000 and will require an average annual expenditure of \$175 for equipment, of \$75 for fertilizer, and of \$750 for current expense. Find an equitable purchase price for the farm on the assumption that money will steadily be worth 5%. How much more could one afford to pay for the farm on the assumption that the "unearned increment" in this neighborhood will amount to 10% in the next ten years?

85. The initial cost of an auto truck is \$3500, its probable life is five years, and its scrap value is \$500. If money is worth 5% effective, what should be the value of this truck at the end of three years?

86. If, in the preceding problem, the upkeep of the machine and the driver's wages amount to \$100 per month, would it be more profitable for a firm to buy and maintain its own truck or to rent one at \$150 per month? (Solve by inspection.) Find the total monthly charge for maintaining this truck.

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87. Two music houses sell their wares for a small cash payment and monthly payments for the balance due. One charges interest at 6%, to be paid on each partial payment at the time that installment is paid; the other charges interest on the total amount outstanding at the time each monthly installment is paid. Show that one is getting simple interest at 6% and the other interest at 6% nominal, convertible monthly. Suppose a dealer to have outstanding accounts for a piano which he sells for \$350, the terms being \$50 cash on day of sale, then \$10 per month and interest until the balance is paid. Assume payments to be immediately reinvested under the same terms, and find the effective rate of interest realized on the loans under the second plan.

NOTE. By a somewhat laborious computation it can be shown that this dealer would realize 5.95% effective on his loans if he made his sales under the first plan.

88. An irrigation company has the following plant: a building with a wearing value of \$30,000 and a probable life of fifty years; flumes and ditches with a total wearing value of \$20,000 and life of twenty years; two motor generators, one costing \$5000 and the other \$8000, with lives of fifteen and twenty years respectively and scrap values of \$250 each; three hydraulic pumps, each having a wearing value of \$500 and a probable life of five years. What amount must be set aside each year to provide for depreciation on a 4% basis? What is the composite life of the plant?

89. A public-utilities corporation issues bonds for \$100,000, bearing interest at 5% payable annually, and redeemable at par in ten years. Is it better for the company to pay the interest from year to year and accumulate a sinking fund at 4% to retire the bonds at maturity, or to buy in the open market at 103 a sufficient number of bonds each year to make the charges for interest and bond retirement approximately the same for each of the ten years? Prepare a schedule

showing the condition of the debt at the end of each year under the second plan.

Hint. The second plan is equivalent to the amortization of a debt of \$103,000, with interest at rate $5/103$ effective, by a series of ten annual payments as nearly equal as possible (see Skinner, pp. 125, 145).

90. Compare the problem in Skinner, p. 145, with the amortization by equal payments for n years of a debt of $K(1+k)$ bearing interest at rate $i/(1+k)$ effective. Hence derive anew formula 11, p. 147.

91. A student during his college course contracts debts amounting to \$2000. He wishes to pay these debts in five equal annual installments, the first payment to be made two years from date of graduation. If he must pay interest at 6% effective, find the annual payment.

92. A charitable organization receives a bequest of \$5000, with the proviso that it be used during the ensuing ten years. What fixed annual income for the ten-year period will this provide if funds can be invested at 5%?

93. A lumber company which has standing timber enough to last for ten years at the present rate of cutting is clearing \$10,000 net each six months from its logging operations. If the net income may be expected to continue at this rate until the timber is cut off, and if the cleared land will then be worth \$10,000, at what price can the company afford to dispose of its holdings? Assume money worth 6%, convertible semiannually.

Also solve this problem on the assumption that because of the increasing scarcity of lumber, the net income increases uniformly 1% each six months over the income for the preceding period.

94. On October 1, 1915, a \$100 bond redeemable at par on October 1, 1920, bearing interest at 5% per annum, payable October 1 and April 1, is sold for \$98. What rate of income does the investor receive? What would be the price to yield the purchaser 6%, convertible October 1 and April 1?

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95. An automobile suitable for doing the hauling required by a certain firm will cost \$2000. Upkeep and wages will cost \$110 per month. It will be desirable to renew the equipment (at the initial cost) at the end of one year, when the car to be replaced can be sold for \$800. If money is worth 6%, what could the prospective buyer afford to pay to have the required work done by others instead of buying and maintaining its own equipment?

96. A typewriter is offered for sale for \$5 in cash and payments of \$3.50 at the end of each month for twenty-four months, or for \$72.50 in cash. The expense of bookkeeping and of insurance against loss on the account may fairly be set at \$2.50, payable when the account is opened. What nominal rate of interest, convertible monthly, is being charged for credit?

97. Show that a first approximation to the rate of interest borne by an annuity is given by

$$i = \frac{2}{n(n-1)}(K/R - n), \text{ or } i = \frac{2}{n(n+1)}(n - A/R).$$

Hint. Solve the equations $K = Rs_{\overline{n}|i}$ and $A = Ra_{\overline{n}|i}$ each for i after replacing any power of $1+i$ by the first three terms of its binomial expansion.

To get a more accurate result, take this approximation and proceed as indicated in Skinner, p. 109. The formulas written above are not sufficiently accurate to use without some further check.

98. Show that a first approximation to the rate of income per period, received by the investor in a bond redeemable at par in n periods, is the dividend rate per period, diminished by one n th part of the ratio of the premium to the par value.

99. A man pays a semiannual premium of \$19.42 for twenty-five years on an endowment bond which pays him \$1000 at the end of the period and provides him with life insurance to the amount of \$1000 during the life of the bond. What sum

deposited at the beginning of each six months for the twenty-five-year period, in a savings bank that pays 3% nominal convertible semiannually, would have accumulated to the same amount? On this basis, how much of each payment on the endowment bond should therefore be considered as the cost of the insurance?

100. A corporation issues \$100 bonds to the amount of \$250,000 on January 1, 1910, redeemable at its option at \$105 on any dividend date after January 1, 1915. The bonds pay interest at 5% per annum. On January 1, 1918, a sinking fund of \$52,500 which has been accumulated is used for bond retirement, and it is desired to retire the remainder of the issue in ten years by payments which, including interest on bonds still outstanding, shall be as nearly equal as possible. Prepare a schedule giving the condition of the debt on each dividend date from January 1, 1918, to January 1, 1928.

ANSWERS

1. 1,209,250,000 tons.
3. \$133.82.
5. 36.73%.
6. \$607.75.
7. \$212.23.
8. Bank pays \$406.04 annually if interest is drawn only at end of each year; bonds pay \$415 annually.
9. 7.4%.
10. \$573.93.
11. 5.25%; 5.2%.
12. \$535.80.
13. 6%; 6.13636%.
14. 24.49%.
15. \$1037.34.
16. 2 years, 3 months, 22 days, by exact method; 2 years, 4 months, by ordinary rule.
17. \$686.18.
18. 42.57609%.
19. \$1496.44.
21. \$442.92.
22. 8 years, 24 days.
23. First plan \$229.32 better.
24. \$9994.11.
25. \$89.38.
26. \$401.21.
29. \$584.84; \$2201.78.
30. \$895.91.
31. In 6.652 years.
32. \$17,686.80.
33. \$7358.17; \$69,057.06.
34. The annuity.
37. \$4500.
39. A slight financial advantage with pavement costing \$2, to be balanced against more frequent interruption of traffic.
42. \$9803.92.
43. \$2910.24.
44. \$127.07.
46. \$1116.81.
47. \$91.341.
48. First plan.
49. \$31,647.38.
50. \$573.16; \$256.59 semiannually, better for debtor.
51. \$708.48 larger; 6.37%.
52. \$49,036.55.
53. \$3.98.
54. \$94,222.38.
55. \$80,637.10; \$16,716.34; yes, \$7967.24 semiannually.
57. \$68.87; \$4.44.
59. \$991.44.
60. \$6740.103; 7.58 years.
61. 7.808 years.
62. \$108.743; \$1850.30; \$1841.57.
63. \$1395.046; \$5407.83; yes, \$641.47 semiannually.
64. \$10.88.
65. \$3.47.
66. 12.28 years.
67. \$407,942.59.
68. \$11,084.63.
69. \$351.69; \$341.45.

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- | | |
|--|---|
| 70. \$148.53 ; \$80.74. | 83. 15.12%. |
| 71. 3.64% nominal, converted
semiannually. | 84. \$20,000 ; \$1227.83. |
| 72. \$432.77. | 85. \$1788.43. |
| 73. 8.20% to 9.7%. | 86. To rent ; \$158.465. |
| 74. To rent. | 87. 6.16778%. |
| 75. 11 years, 3 months, 16 days ;
\$292.59. | 88. \$1642.56 ; 23.9 years. |
| 76. \$2140.04 ; \$712.98. | 89. Buy at 103. |
| 77. \$93.554. | 91. \$503.28. |
| 78. \$26,998.60. | 92. \$647.52. |
| 79. 10 years. | 93. \$154,311.51 ; \$167,741.96. |
| 80. \$16.04. | 94. 5.5% nominal, converted semi-
annually ; \$95.734. |
| 81. 1 year, 9 months. | 95. \$217.09 per month. |
| 82. \$2353.98. | 96. 18.16%. |
| | 99. \$13.37 ; \$6.05. |

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